1.5 Inverse Functions & "u" Substitution with Logarithms Notes

Find the inverse of each function.

a. $f(x)=\ln(x+2)-3$

b. $f(x)=\log_3(2x+1)+5$

c. $f(x)=5^{x-6}+1$

d. $f(x)=5\cdot 2^{3-x}-4$

a.	
$y=\ln(x+2)-3$	Substitute each x with y and y with x.
$x=\ln(y+2)-3$	Isolate the logarithmic term.
$x+3=\ln(y+2)$	
$e^{x+3}=e^{\ln(y+2)}$	Use the property $e^{\ln x}=x$ to eliminate the logarithm.
$e^{x+3}=y+2$	
$e^{x+3}-2=y$	Solve for y.
$e^{x+3}-2=f^{-1}(x)$	

b.	
$y=\log_3(2x+1)+5$	Substitute each x with y and y with x.
$x=\log_3(2y+1)+5$	Isolate the logarithmic term.
$x-5=\log_3(2y+1)$	
$3^{x-5}=3^{\log_3(2y+1)}$	Use the property $b^{\log_b x}=x$ to eliminate the logarithm.
$3^{x-5}=2y+1$	
$3^{x-5}-1=2y$	Solve for y.
$\frac{3^{x-5}-1}{2}=y$	
$\frac{3^{x-5}-1}{2}=f^{-1}(x)$	

Now you try: Find the inverse of $f(x)=\log(x+7)-2$

c.	
$y = 5^{x-6} + 1$ $x = 5^{y-6} + 1$ $x - 1 = 5^{y-6}$ $\log_5(x-1) = \log_5(5^{y-6})$ $\log_5(x-1) = y - 6$ $\log_5(x-1) + 6 = y$ $\log_5(x-1) + 6 = f^{-1}(x)$	Substitute each x with y and y with x. Isolate the exponential term. Use the property $\log_b b^x = x$ to eliminate the base of the exponent. Solve for y.

d.	
$y = 5 \cdot 2^{3-x} - 4$ $x = 5 \cdot 2^{3-y} - 4$ $x + 4 = 5 \cdot 2^{3-y}$ $\frac{x+4}{5} = 2^{3-y}$ $\log_2 \frac{x+4}{5} = \log_2 2^{3-y}$ $\log_2 \frac{x+4}{5} = 3 - y$ $\log_2 \frac{x+4}{5} - 3 = -y$ $-\log_2 \frac{x+4}{5} + 3 = y$ $3 - \log_2 \frac{x+4}{5} = f^{-1}(x)$	Substitute each x with y and y with x. Isolate the exponential term. Use the property $\log_b b^x = x$ to eliminate the base of the exponent. Solve for y.

Find the inverse of: $f(x) = e^{4x-5} - 7$

Using the Structure of Expressions to Solve Equations

Example

Solve the equation $e^{4x} - 3e^{2x} - 18 = 0$.

$e^{4x} - 3e^{2x} - 18 = 0$	
$u^2 - 3u - 18 = 0$ $(u-6)(u+3) = 0$ $u-6=0$ $\quad$ $u+3=0$ $u=6$ $\quad$ $u=-3$	The equation is quadratic in nature, let $u = e^{2x}$. Rewrite the equation in terms of u. Solve for u.
$e^{2x} = 6$ $\quad$ $e^{2x} = -3$ $\ln e^{2x} = \ln 6$ $2x = \ln 6$ $x = \frac{\ln 6}{2}$	Substitute $e^{2x} = u$ and solve for x. An exponential function will never equal a negative number.

You try. Solve the equations using u-substitution.

$e^{2x} - 2e^{x} - 3 = 0$

$5^{2x} + 5^{x} - 6 = 0$